The Postulates of Quantum Mechanics

POSTULATE I. The state of a single particle is described as fully as possible by an appropriate state function, $\Psi(x, y, z, t)$, which may be expressed for stationary energy states as the product of a time-independent amplitude function, $\psi(x, y, z)$ and a time-dependent function $f(t)$. Both functions, Ψ and ψ, must be single-valued, continuous, and finite for all values of their coordinates, and must be smoothly varying within the boundaries at which they go to zero.

III

POSTULATE ~~II~~. a) *The possible state functions $\Psi(x, y, z, t)$ for a single particle are given by the solution of the time-dependent Schrödinger wave equation:*

$$\left[\frac{-\hbar^2}{2m}\nabla^2 + V(x, y, z, t)\right]\Psi = \left[\frac{-\hbar}{i}\left(\frac{\partial}{\partial t}\right)\right]\Psi,$$

b) *In the special case of conservative systems, where V is not dependent on t, and $\Psi(x, y, z, t) = \psi(x, y, z)e^{-iEt/\hbar}$, the possible time-independent amplitude functions $\psi(x, y, z)$ for a single particle are given by solution of the time-independent Schrödinger equation,*

$$\left[\frac{-\hbar^2}{2m}\nabla^2 + V(x, y, z)\right]\psi = E\psi,$$

II

POSTULATE ~~III~~. a) *For every dynamical variable there is a corresponding linear operator $\hat{A}$. If the dynamical variable is capable of exact experimental determination, its only possible exact values A are those given by the eigenvalues of the equation*

$$\hat{A}\Psi = A\Psi.$$

b) *In the special case of conservative systems and for time-independent operators, the only possible exact values of the dynamical variable are also given by*

$$\hat{A}\psi = A\psi.$$

POSTULATE IV. a) *The term $|\Psi|^2\, d\tau$ or $\Psi^*\Psi\, d\tau$ is the time-dependent probability that a single particle exists at a given time t in the space element $d\tau$, that is, between x, y, z and $(x + dx)$, $(y + dy)$, and $(z + dz)$.*

b) *For the special case of a conservative system, in which the single particle is restricted to an energy eigenstate, the time-independent probability that the particle exists in the space element $d\tau$ is $|\psi|^2\, d\tau$ or $\psi^*\psi\, d\tau$.*

POSTULATE V. a) *The expected mean value $\bar{A}$ of a series of measurements of an observable A made over a large number of particles each in the state represented by Ψ is*

$$\bar{A} = \frac{\int_{-\infty}^{+\infty} \Psi^*\hat{A}\Psi\, d\tau}{\int_{-\infty}^{+\infty} \Psi^*\Psi\, d\tau},$$

where $\hat{A}$ is the quantum-mechanical operator for the observable.

b) *For the special case of conservative systems in which the operator $\hat{A}$ does not depend explicitly on time,*

$$\bar{A} = \frac{\int_{-\infty}^{+\infty} \psi^*\hat{A}\psi\, d\tau}{\int_{-\infty}^{+\infty} \psi^*\psi\, d\tau}.$$

c) *For the special case of conservative systems and time-independent operators and where the wave functions are normalized,*

$$\bar{A} = \int_{-\infty}^{+\infty} \psi^*\hat{A}\psi\, d\tau.$$

PAULI EXCLUSION PRINCIPLE. To be acceptable, the total wave function for a system of electrons must be antisymmetric to the simultaneous exchange of coordinates and spins between any pair of electrons.