TERM TEST 2

QUESTION 1

Same as 4-16 (b)

$\hat{A} = \frac{d}{dx} - x \qquad \hat{B} = x + \frac{d}{dx} \qquad [\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$

$$\hat{A}\hat{B} f = \left(\frac{d}{dx} - x\right)\left(x + \frac{d}{dx}\right) f = \left(\frac{d}{dx} - x\right)\left[xf + \frac{df}{dx}\right]$$

$$= \frac{d}{dx}\left[xf + \frac{df}{dx}\right] - x\left[xf + \frac{df}{dx}\right]$$

$$= x\frac{df}{dx} + f + \frac{d^2f}{dx^2} - x^2 f - x\frac{df}{dx}$$

$$= \frac{d^2f}{dx^2} + f - x^2 f$$

$$\hat{B}\hat{A} f = \left(x + \frac{d}{dx}\right)\left(\frac{d}{dx} - x\right) f = \left(x + \frac{d}{dx}\right)\left[\frac{df}{dx} - xf\right]$$

$$= x\frac{df}{dx} - x^2 f + \frac{d^2f}{dx^2} - x\frac{df}{dx} - f$$

$$= \frac{d^2f}{dx^2} - f - x^2 f$$

$\hat{A}\hat{B} f - \hat{B}\hat{A} f = 2f \qquad \therefore [A, B] = 2$ Not commute

QUESTION 2

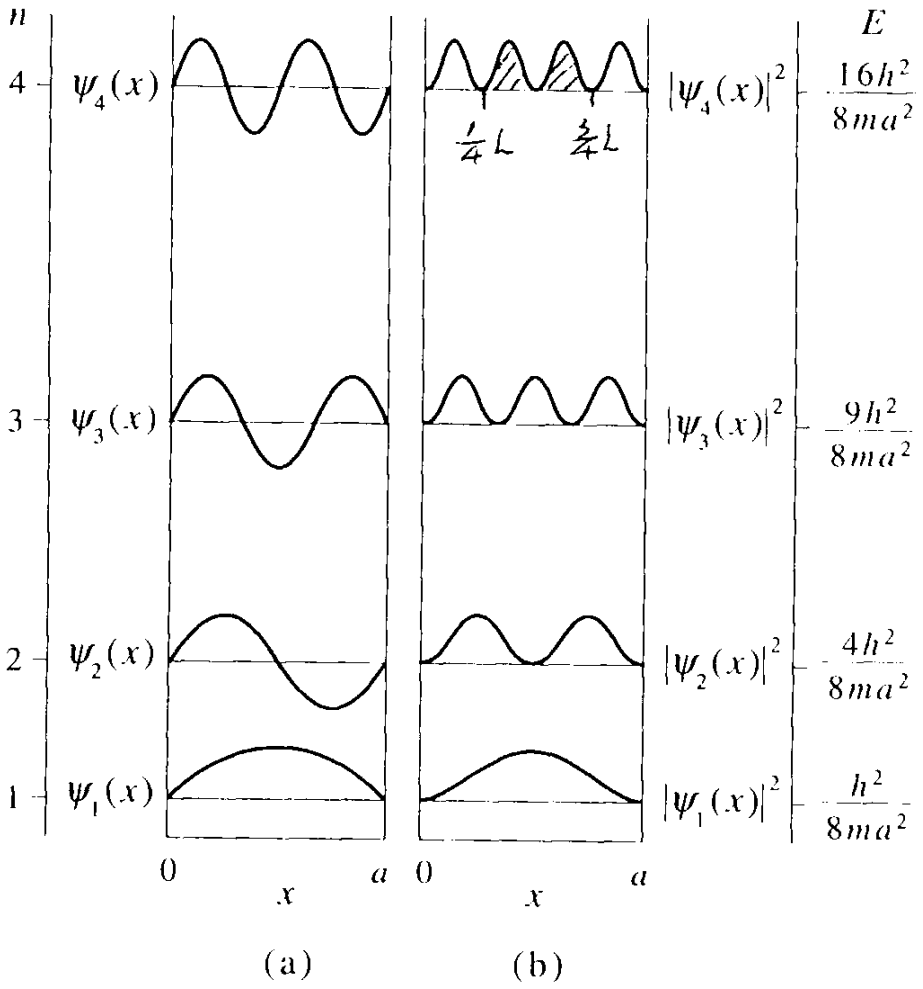
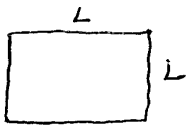


FIGURE 3.2
The energy levels, wave functions (a), and probability densities (b) for the particle in a box.

From Figure 3.2 of McQuarrie (L=a), we see that the probability densities function, the area under the curve between 1/4L and 3/4L is exactly half that of between 0 and L. So, the answer is 1/2.

QUESTION 3



(a) $\hat{H}\psi = E\psi$

$(\hat{T} + \hat{V})\psi = E\psi$

where $\hat{T} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)$

$$\hat{V} = \begin{cases} 0, & 0 < x < L \text{ and } 0 < y < L \\ \infty, & \text{otherwise} \end{cases}$$

$\therefore$ For $\hat{V} = \infty$, $\psi = 0$ (Outside the square)

For $\hat{V} = 0$, we have

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\psi_{n_x n_y} = E_{n_x n_y}\psi_{n_x n_y} \quad \text{(Inside the box)}$$

$$E_{n_x n_y} = \frac{h}{8mL^2}(n_x^2 + n_y^2)$$

(b)

$\psi_{n_x n_y}$	n_x	n_y	$E_{n_x n_y}$ ($h^2/8mL^2$ as the unit)	Degeneracy
$\|1,1\rangle$	1	1	2	1
$\|1,2\rangle$	1	2	5	2
$\|2,1\rangle$	2	1		
$\|2,2\rangle$	2	2	8	1
$\|1,3\rangle$	1	3	10	2
$\|3,1\rangle$	3	1		
$\|2,3\rangle$	2	3	13	2
$\|3,2\rangle$	3	2		
$\|1,4\rangle$	1	4	17	2
$\|4,1\rangle$	4	1		

QUESTION 4

Examples of manifestation of quantum effects are (any two of the following):

- Zero-point vibrational energy
- Tunnelling
- Uncertainty relation
- Wave-particle duality
- Spin