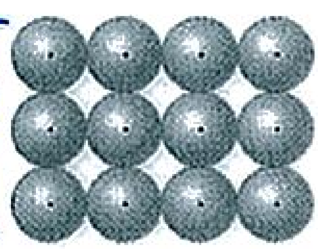
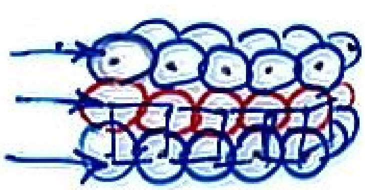
A. Simple cubic packing

layer 1 (looking down from above)



The sphere centres are aligned to form rows & columns

Subsequent layers are added by lining up spheres with those in the layer below.

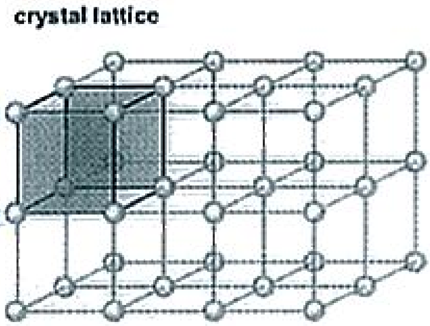


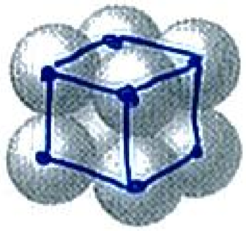
$\frac{1}{8}$ of a sphere

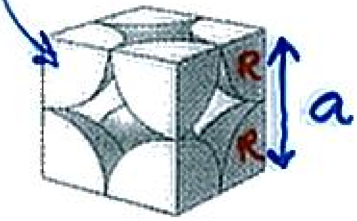
layer 3 →

layer 2 →

layer 1 →







unit cell

How many spheres are there per unit cell? What is the $\overbrace{\text{edge length}}^{a}$ and volume of the unit cell?

What is the packing efficiency? how much of cube is not empty space

$$\underset{\text{(per cell)}}{\text{\# spheres}} = \frac{1}{8}\ \text{sphere/corner} \times 8\ \text{corners} = 1$$

edge length? $a = 2R$

$$V_{cell} = a^3 = (2R)^3 = 8R^3$$

volume of a sphere

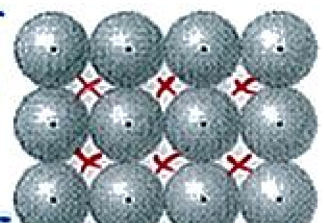
$$\text{Packing efficiency}: \quad \epsilon = \frac{V_{spheres}}{V_{cell}} = \frac{1 \times \frac{4}{3}\pi \cancel{R^3}}{8\cancel{R^3}}$$

$$= \frac{\pi}{6} = 0.5236$$

$\therefore \approx 48\%$ of the unit cell is empty space for sc packing arrangement.

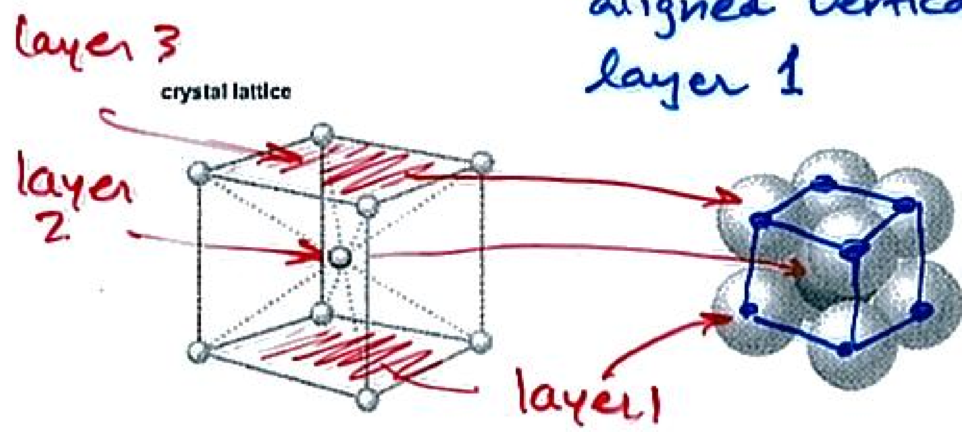
B. Body-centred cubic packing

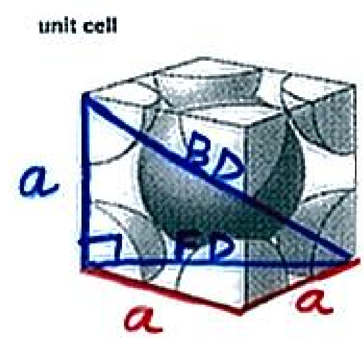
layer 1 - looking down from above



Spheres of layer 2 are placed in the "dimples" (×) of layer 1, and push downwards until the spheres of layer 1 are pushed apart.

For layer 3, the spheres are aligned vertically with spheres in layer 1

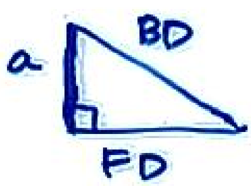




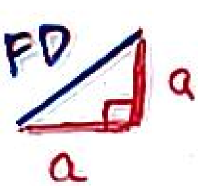
$$\#\text{ spheres (per cell)} = \frac{1}{8} \times 8 + 1 \text{ sphere at centre} = 2$$

BD = body-diagonal $= R + 2R + R = 4R$

FD = face-diagonal



$$a^2 + FD^2 = BD^2$$



$$a^2 + a^2 = FD^2$$

$$\therefore 2a^2 = FD^2$$

$$\Rightarrow a^2 + 2a^2 = BD^2$$

$$3a^2 = BD^2$$

$$a = \frac{1}{\sqrt{3}} BD$$

$\therefore\ a = \frac{1}{\sqrt{3}}(4R) = \frac{4}{\sqrt{3}} R = 2.3\ R$

Note: unit cell for bcc packing is larger than that of sc

Packing efficiency ? $\quad \epsilon = \dfrac{2 \times \frac{4}{3}\pi R^3}{\left(\frac{4}{\sqrt{3}} R\right)^3}$

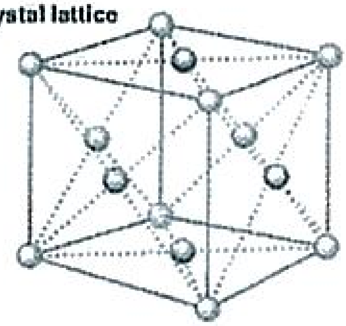
$$= \frac{\frac{\cancel{8}\pi}{\cancel{3}}}{\frac{8\,\cancel{64}}{\cancel{3}\sqrt{3}}} = \frac{\sqrt{3}\,\pi}{8}$$

$$= 0.6802$$

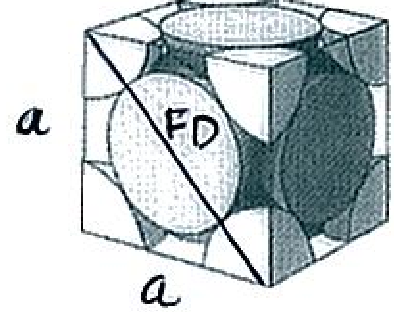
$\therefore \simeq 32\%$ of unit cell is empty space.

C. Face-centred cubic packing

crystal lattice



unit cell



$$\#\text{spheres (per cell)} = \frac{1}{8}\,\frac{\text{sphere}}{\text{corner}} \times 8 \text{ corners} + \frac{1}{2}\,\frac{\text{sphere}}{\text{face}} \times 6 \text{ faces}$$

$$= 1 + 3 = 4$$

edge length?

$$a^2 + a^2 = FD^2$$
$$2a^2 = FD^2$$
$$a = \frac{1}{\sqrt{2}} FD$$

But $FD = 4R$

$$\therefore\ a = \frac{4}{\sqrt{2}} R$$
$$a = 2\sqrt{2}\,R$$
$$\approx 2.8\,R$$

Packing efficiency?

$$\epsilon = \frac{V_{spheres}}{V_{cell}} = \frac{4 \times \frac{4}{3}\pi R^3}{(2\sqrt{2}R)^3}$$

$$= \frac{\frac{\cancel{16}\pi}{3}}{\cancel{16}\sqrt{2}} = \frac{\pi}{3\sqrt{2}} = 0.7405$$

26% of the fcc unit cell is empty space.

We'll see shortly that this is the maximum packing efficiency possible when packing identical spheres.